C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name: Problem Solving-I

Subject Code: 5SC02PRS1   **Branch: M.Sc. (Mathematics)**

Semester: II   **Date: 27/04/2018**   **Time: 10:30 To 01:30**   **Marks: 70**

Instructions:
    (1) Use of Programmable calculator and any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

  a. Find $\operatorname{Im}\left(\dfrac{1}{z}\right)$, where $z = \left(\sin x + i\cos x\right)^{7}$.   **(02)**

  b. Find the rank of $\begin{bmatrix} -1 & 2 & 3 \\ 2 & -4 & -6 \\ 3 & -6 & 9 \end{bmatrix}$.   **(02)**

  c. Give an example of function have removable singularity.   **(01)**
  d. Determine the function $f\left(z\right)$ for which $f\left(0\right) = 0$ further $f\left(z\right)$ is bounded and $f'\left(z\right)$ **(01)**
      is continuous.
  e. Write the orthonormal basis of M_{22}.   **(01)**

Q-2 Attempt all questions   **(14)**
  a. For the Strum-Lioville problem $X'' + \lambda X = 0,\, X\left(0\right) = X\left(\pi\right),\, X'\left(0\right) = X'\left(\pi\right)$ **(07)**
      obtain the eigenfunctions and the corresponding eigenvalues.

  b. Solve: $\dfrac{dy}{dx} + 2y\tan x = \sin x$; $y\left(\dfrac{\pi}{3}\right) = 0$ **(04)**

  c. Evaluate: $\displaystyle\oint_{c} \dfrac{z^{4}+1}{\left(z-1\right)^{2}}\, dz$ where C is the circle i) $|z| = \dfrac{1}{2}$ and ii) $|z| = 2$ **(03)**

OR

Q-2 Attempt all questions   **(14)**

a. Solve: i) $\left(x^2 - y^2\right)dx = xy\,dy$ **(04)**

   ii) $\left(D^3 + 3D^2 + 3D + 1\right)y = e^{-x}$ **(03)**

b. Find harmonic conjugate of $\log\left(x^2 + y^2\right)$.   **(04)**

c. Evaluate: $\lim\limits_{z \to 0} \dfrac{\sin \bar{z}}{z}$ **(03)**

Q-3 Attempt all questions   **(14)**

a. Solve the differential equation $\left(D^2 + 2D + 1\right)y = e^{-x}\log x$ by the method of   **(05)**
   variation of parameter.

b. Evaluate: $\oint\limits_{C} \dfrac{z+1}{\sin z}\,dz$, where $C:|z| = \dfrac{3\pi}{2}$.   **(05)**

c. Check whether the following functions are analytic or not.   **(04)**

   i) $f(z) = z^{\frac{5}{2}}$ ii) $f(z) = \bar{z} + z$

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OR

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Q-3 Attempt all questions   **(14)**

a. Solve: $\left(D^2 - 4D + 3\right)y = \sin 3x \cos 2x$.   **(04)**

b. Is $w = \dfrac{1}{z}$ conformal at $1 + i$?   **(03)**

c. Evaluate: $\oint\limits_{C} \dfrac{z^3 + z^2 + 1}{z(z-1)}\,dz$, where $C:|z| = \dfrac{\pi}{2}$.   **(04)**

d. Show that the map $T : P_2 \to R$ which is defined by $T\left(a + bx + cx^2\right) = a + b + c$ is   **(03)**
   linear transformation.

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SECTION - II

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Q-4 Answer the Following questions: **(07)**

a. Solve: $\left(D^3 + 3D\right)y = 0$ **(02)**

b. Prove that the system of three vectors $(1,3,2), (1,-7,-8)\ \&\ (2,1,-1)$ of $V_3(R)$ is   **(02)**
   linearly dependent.

c. If u and v are orthogonal vectors in M_{22} and $u = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ then find v.   **(01)**

d. What is the span of $\{1, t, 3t\}$?   **(01)**

e. Solve: $y' - y = \dfrac{1}{e^x}$ **(01)**

Q-5 Attempt all questions (14)

a. Find A^{-1} by using Cayley-Hamilton theorem, where $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. (05)

b. Evaluate $\oint_C z^2 \, dz$, where C is the Square whose boundaries are $x = \pm a$ and $y = \pm a$. (05)

c. Solve: $2y'' + 8y = 6\tan(3t)$ (04)

OR

Q-5 Attempt all questions (14)

a. Solve the following system of linear equation: (05)
$x + y - z = 1, \; -x - y + 5z = -1, \; 3x + 5y + 7z = 2$

b. Using Cauchy's integral formula, evaluate $\oint_C \dfrac{e^{2z}}{z^2 - 3z + 2} \, dz,$ where C is $|z| = 3$. (05)

c. Find bilinear transformation which sends the points i , 0 , 1 into –i ,1,-1 respectively. (04)

Q-6 Attempt all questions (14)

a. Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (07)

b. If $V = M_{22}$ then show that V is inner space, where $\langle A, B \rangle = tr(A^t B)$. (05)

c. If $V = Q$ and operations are usual then show that V is not a vector space over R. (02)

OR

Q-6 Attempt all Questions (14)

a. Using gram-Schmidt orthogonalization process find orthogonal basis from the (07)
basis $B = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \right\}$.

b. Show that $\{(1,-1,2),(-1,2,2),(1,0,1)\}$ is basis of R^3. (03)

c. Which of the following are subspace? Justify with proper reason. (04)

    1) $W_1 = \{(x, y) | x + y = e^x\}$

    2) $W_2 = \{(x, y, z) | x + y - z = 0\}$